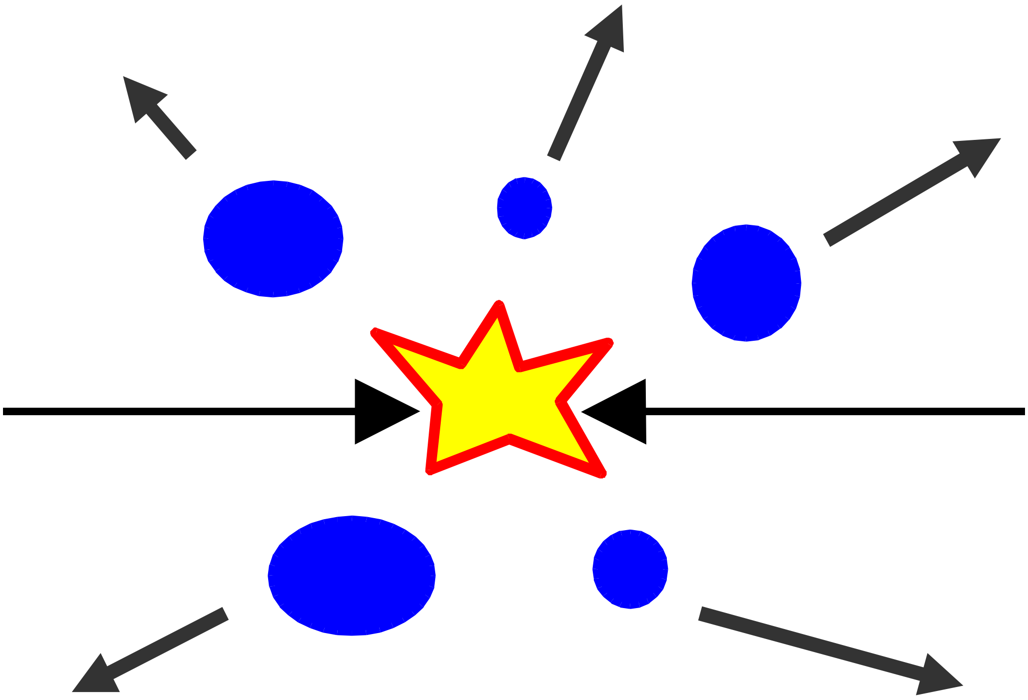
Chemical equilibrium of Strangeness

Outline

- Sketch of the statistical model
- Statistical model analysis and Strangeness production in heavy ion collisions from 1A GeV/c to 160A GeV/c + RHIC
- Technical issues on fits
- Conclusions



STATISTICAL HADRONISATION MODEL

i. The final result of any high energy collision is the formation of a set of pre-hadronic clusters (fireballs) having a volume, a four-momentum and a set of internal charges (electric, strangeness etc..)

ii. Somehow clusters inherit their relevant physical quantities by previous dynamical evolution; their distribution may vary with type of collision and centre-of-mass energy

iii. *Each cluster gives rise to hadrons according to the Gibb's law of statistical mechanics, i.e. Every multihadronic state compatible with conservation laws is equally likely*

UNIVERSALITY OF HADRONISATION PROCESS

Local (= single cluster) statistical equilibrium is not enough to calculate physical observables. Also the probabilities of cluster configurations must be known

$$N \quad \{(P_{1,} \mathbf{Q}_{\mathbf{1},} V_1),\ldots,(P_{N,} \mathbf{Q}_{\mathbf{N},} V_N)\}$$

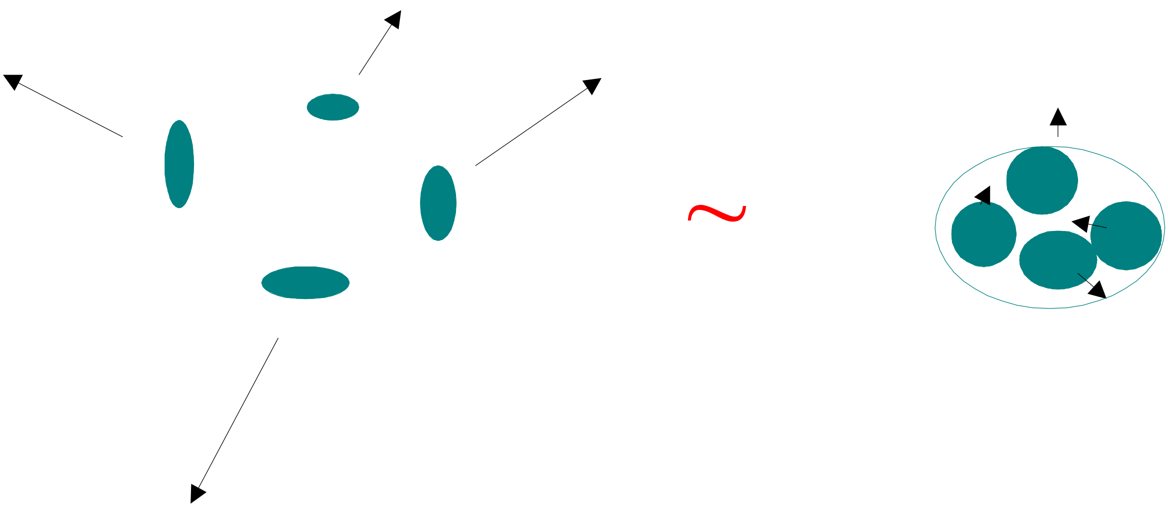
By using the conditional probability decomposition:

$$\langle O \rangle = \sum_N P_N \sum_{\{P_{i,} \mathbf{Q}_{\mathbf{i},} V_i\}} f(\{P_{i,} \mathbf{Q}_{\mathbf{i},} V_i\}) \sum_{i=1}^{N} \langle O \rangle_i$$

P_N and f determined by dynamical evolution

If O is a Lorentz scalar, then single-cluster averages depend only on P^2_i rather than P_i and Lorentz transformations on individual clusters leave the overall average $\langle O \rangle$ unchanged.

This can be taken advantage of to prove the equivalence between the actual distribution of four-momenta and a suitable distribution, i.e. that obtained by dividing one cluster into N parts (reduction to an *equivalent global cluster*, EGC)



For the equivalence to hold, a further crucial assumption is necessary:

the actual conditional probability distribution of masses and charges $g(\{M_i,\mathbf{Q}_i\}|\ V^*_i)$ with fixed proper volumes must be the same as that associated to the splitting of the EGC into N clusters

If this *non-trivial hypothesis* is true, then: (F. B., G. Passaleva hep/ph 0107xxx)

$$\langle O\rangle=\sum_N \sum_{\{P_{i,}\mathbf{Q}_{i,}V_i\}} f(\{P_{i,}\mathbf{Q}_{i,}V_i\})\sum_{i=1}^{N}\langle O\rangle_i=$$
$$=\int dV\int dM\,\chi(M,V)\langle O\rangle_{(V,M,\mathbf{Q})}$$

where *M* and *V* are the EGC's mass and volume, $\mathbf{Q}=\sum\mathbf{Q}_i$ e χ an input arbitrary distribution.

If *M* and *V* are large, it is possible to use a canonical approximation of the above formula:

$$\langle O\rangle=\int dV\int dT\,\zeta(T,V)\langle O\rangle_{(T,V,\mathbf{Q})}$$

EGC has volume and mass much larger than physical clusters and the transition from microcanonical to canonical treatment is certainly easier for it than for each individual cluster

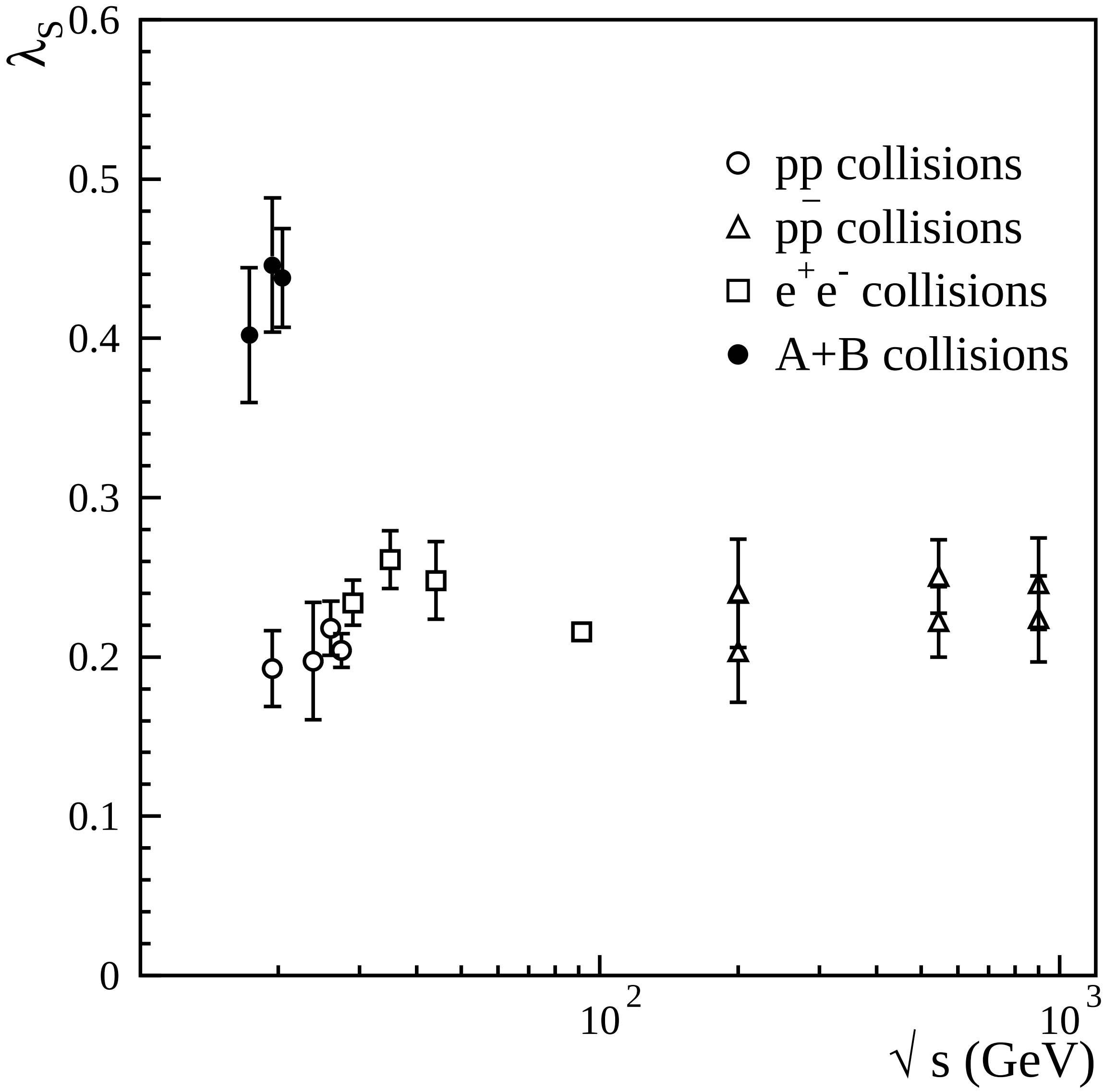
It may well happen that, if EGC exists, canonical ensemble can be used to calculate global averages whereas each cluster are too small for it to apply. Therefore, temperature might be a well-defined concept only in a global sense, i.e. T would be globally but not locally defined!

EXTRA STRANGENESS SUPPRESSION

Particles carrying strange quarks not at complete equilibrium.
The number of newly produced strange quarks per u, d quark is found to be fairly constant in elementary collisions.

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u}\rangle + \langle d\bar{d}\rangle}$$

F.B., M. Gazdzicki, J. Sollfrank Eur. Phys. J. C 5 (1998) 143



NEW PARAMETRISATION OF STRANGENESS SUPPRESSION

"Traditional" parametrisation of strangeness suppression through an additional factor γ_s^n where n is the number of strange quarks.

In view of the constancy of λ_s the free parameter is taken to be *the (mean) number of strange quarks to be hadronised* (coalescence statistical model)

➨ The partition function of the hadron gas should be calculated by fixing the absolute number of valence strange quarks

➨The number of newly created ss pairs is assumed to fluctuate poissonianly (independent creation)

⬇

$$\langle n_j \rangle = \sum_{K=0}^{\infty} \mathrm{e}^{-\langle s\bar{s}\rangle} \frac{\langle s\bar{s}\rangle^K}{K!} \frac{(2J_j+1)V}{(2\pi)^3} \int d^3p \; \exp(-\sqrt{p^2+m_j^2}/T) \frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})}$$

K = number of newly created strange quark pairs

$\mathbf{Q}$ = (Q, B, S, N_s = $2K$ + initial strange quarks)

$\mathbf{q}_j$ = (Q_j, B_j, S_j, N_{sj}) are the charges of the hadron

4-dimensional numerical integration requires unpractically long CPU times

➨ analytical reduction to a 2-dimensional integration is possible (F.B., G. Passaleva, hep-ph 0107xxx)

Calculation scheme

The canonical partition function reads:

$$Z(\mathbf{Q}) = \frac{1}{(2\pi)^4} \int d^4\phi \; \mathrm{e}^{i\mathbf{Q}\cdot\phi} \exp(F(V,T,\phi))$$

$$F(V,T,\phi) = \sum_j \sum_{n=1}^{\infty} \frac{(\mp 1)^{\pm 1}}{n} z_{j(n)} \mathrm{e}^{-in\mathbf{q}\cdot\phi} \qquad z_{j(n)} = \frac{(2J_j+1)V}{(2\pi)^3} \int d^3p \; \mathrm{e}^{-n\sqrt{p^2+m_j^2}/T}$$

- The integration in ϕ_B can be done at once as baryon number can only be 1 or -1 (and not 2, 3,...) *neglecting Fermi statistics*

$$Z(\mathbf{Q}) = \frac{1}{(2\pi)^3} \int d^3\phi \; \mathrm{e}^{i\mathbf{Q}\cdot\phi} \exp(F_{mes}(V,T,\phi)) \sum_{k=0}^{\infty} \frac{W_+^{k+B}(\phi) W_-^k(\phi)}{k!(k+B)!}$$

with

$$W_\pm = \frac{V}{(2\pi)^3} \sum_{\substack{bar.\\ antibar.}} z_{j(1)} \exp(-i\mathbf{q}_j\cdot\phi)$$

- Next integration is performed by setting $e^{-\iota\phi} = w$ and using the residual theorem. Advantage is taken of the analiticity of $F(w)$ as only positive positive powers of w are involved

$$Z(\mathbf{Q}) = \frac{1}{(2\pi)^2} \int d^2\phi \; \mathrm{e}^{i\mathbf{Q}\cdot\phi} \exp(\alpha(\phi)) \frac{1}{N_{S!}} D^{N_s}[S(x)\beta(\phi)x + \gamma(\phi)x^2]_{x=0}$$

$$F_{mes}(w) = \alpha + \beta w + \gamma w^2 \qquad S(x) = I_B(2|W_+(x)|)\exp[i\,B\arg W_+(x)]$$

In practice, it is not possible to calculate the partition function with K > 4 within reasonable CPU time and the method is thus limited to such maximum number of ss pairs

Therefore, the sum over ss pairs is truncated and renormalised accordingly:

$$\langle n_j \rangle = \frac{1}{N_f} \sum_{K=0}^{K_{max}} \frac{\langle s\bar{s}\rangle^K}{K!} \frac{(2J_j+1)V}{(2\pi)^3} \int d^3p \, \exp\left(-\sqrt{p^2+m_j^2}/T\right) \frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})}$$

The cut K=4 is large enough to treat actually measured elementary collisions!

In the grand-canonical limit, this parametrisation is equivalent to the 'traditional' γ_S parametrisation of strangeness suppression. Indeed, γ_S is the fugacity relevatn to the total number of strange quarks (Rafelski 1995, Slotta Sollfrank Heinz 1995)

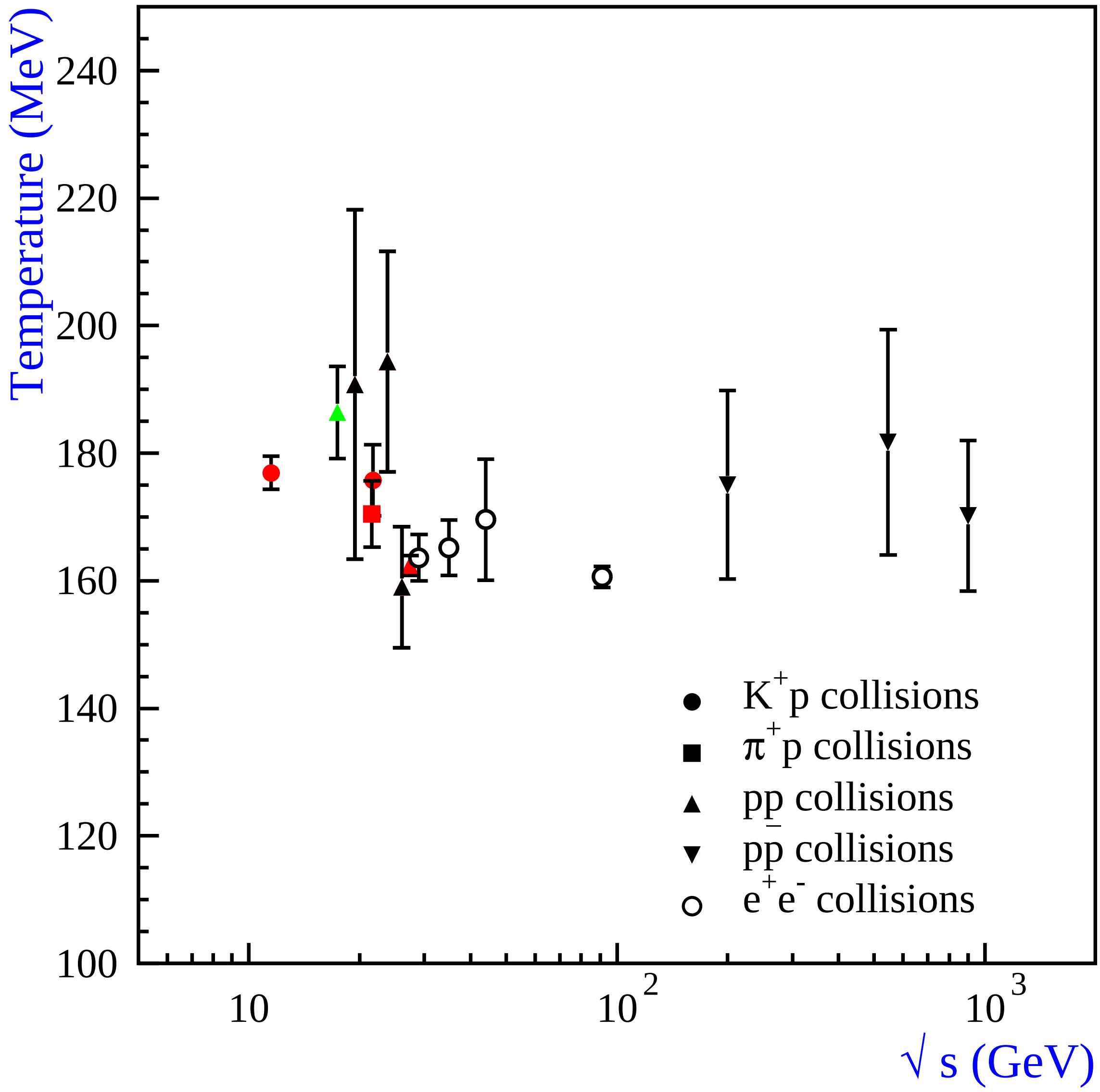
$$\frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})} \to \exp(\mu\cdot\mathbf{q}_j/T) \qquad \exp(\mu_{N_s} N_{Sj}/T) \equiv \gamma_S = \gamma_S(N_S)$$

and

$$\sum_{K=0}^{\infty} \mathrm{e}^{-\langle s\bar{s}\rangle} \frac{\langle s\bar{s}\rangle^K}{K!} \gamma_S(K) \simeq \gamma_S(\langle s\bar{s}\rangle)$$

SUMMARY PLOTS OF FITS TO ELEMENTARY COLLISIONS

- New fits with <ss> (F.B., G. Passaleva, hep-ph 0107xxx)
- Preliminary fit to NA49 pp data (F.B., R. Stock)
- Old data (F.B. Proc. XXXIII ELN Workshop, F.B. U. Heinz Z. Phys. C 76 (1997), 269)



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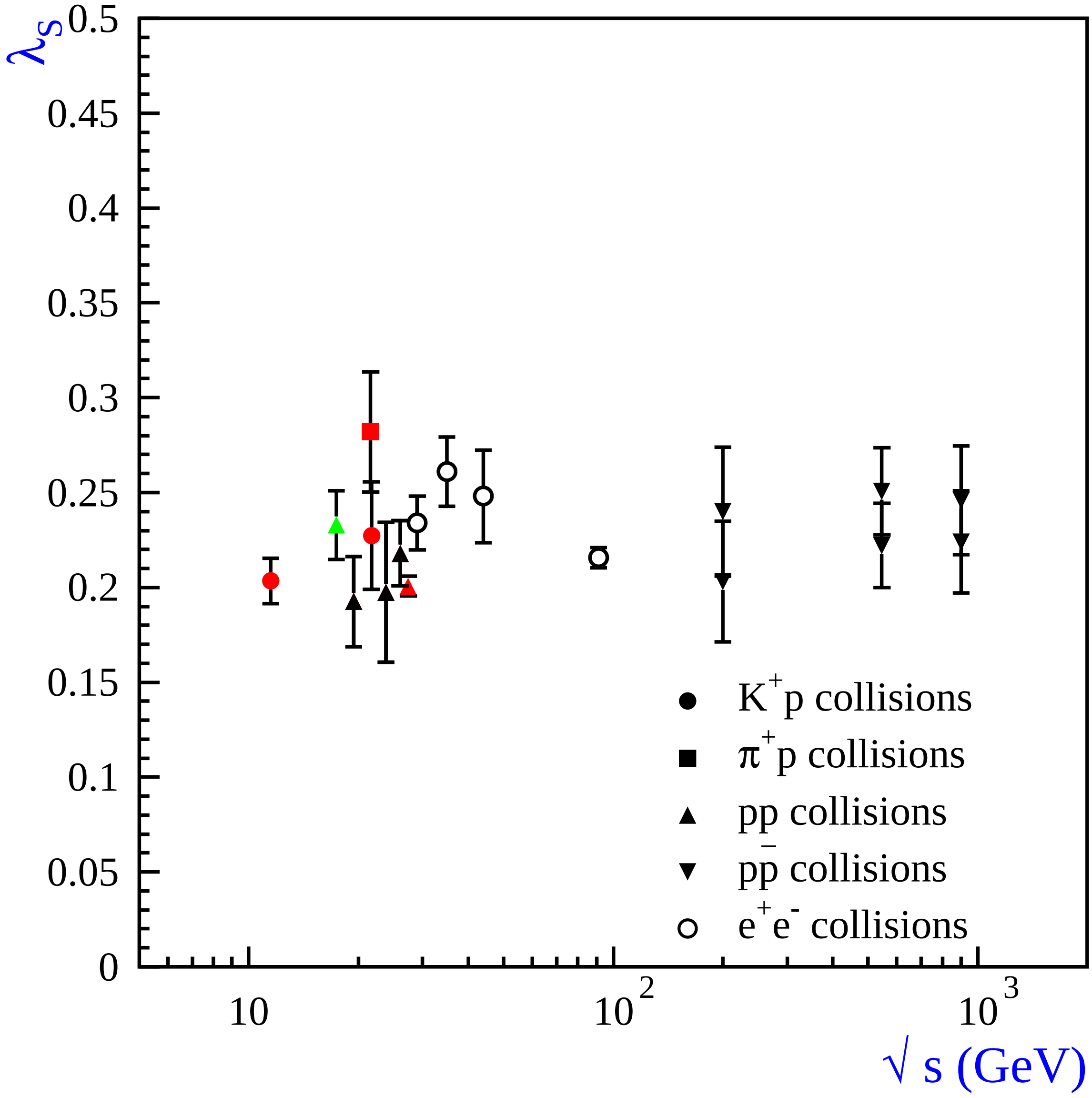


HEAVY ION COLLISIONS FROM SIS TO SPS

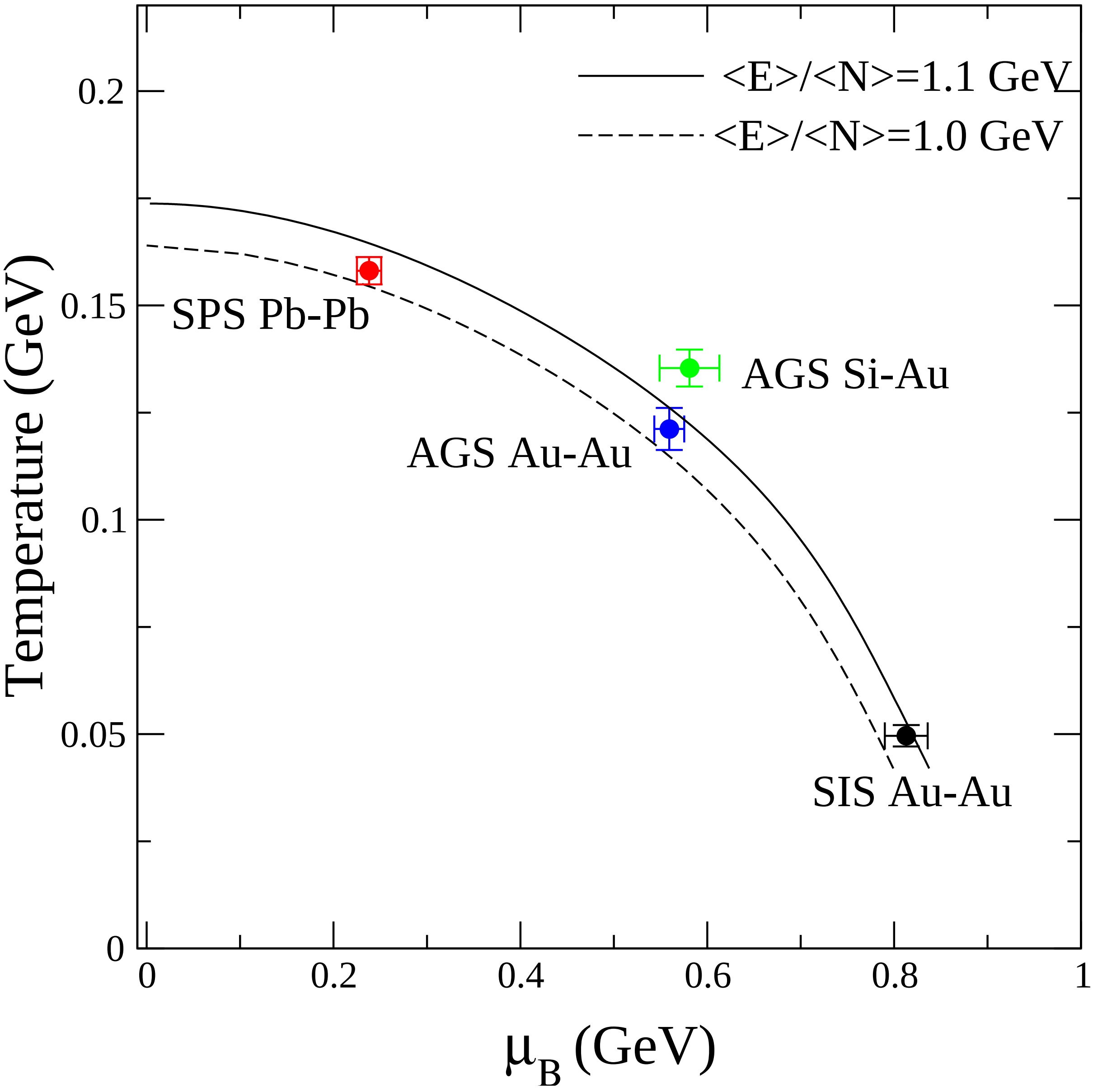
F.B., J. Cleymans, A. Keranen, E. Suhonen, K. Redlich

Phys. Rev. C 64 024901, 2001

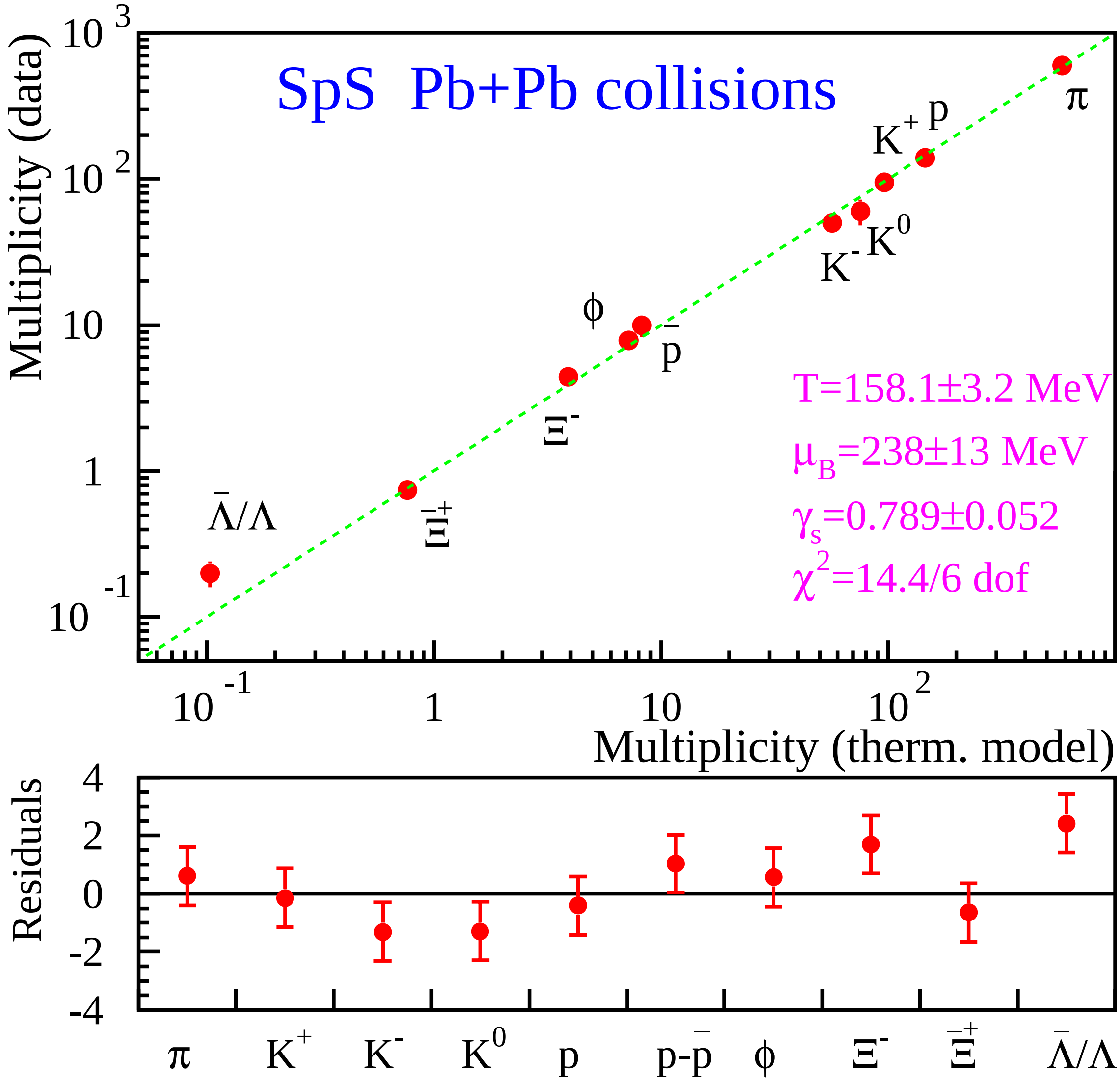
- Fits to 4π multiplicities and multiplicity ratios (except 2 ratios in Si-Au) measured or extrapolated
- In Pb-Pb @ SPS, all of 4π measurements come from NA49
- Grand-canonical ensemble used in Pb-Pb @ SPS and Au-Au @AGS whilst strangeness-canonical ensemble used in Si-Au @ AGS and SIS (S=0 exact, requiring 4π data)
- Two completely independent programs allowing to cross-check with each other and yielding outcomes in satisfactorily agreement

In Pb-Pb, the updated Ξ multiplicity measurement by NA49 entailed a significant lowering of fitted temperature from ~ 180 to ~ 160 MeV, i.e. the same value found in the most accurate fits in elementary collisions at high energy

TEMPERATURE AND BARYOCHEMICAL POTENTIAL



FIT TO NA49 4π data PbPb @ 160 GeV

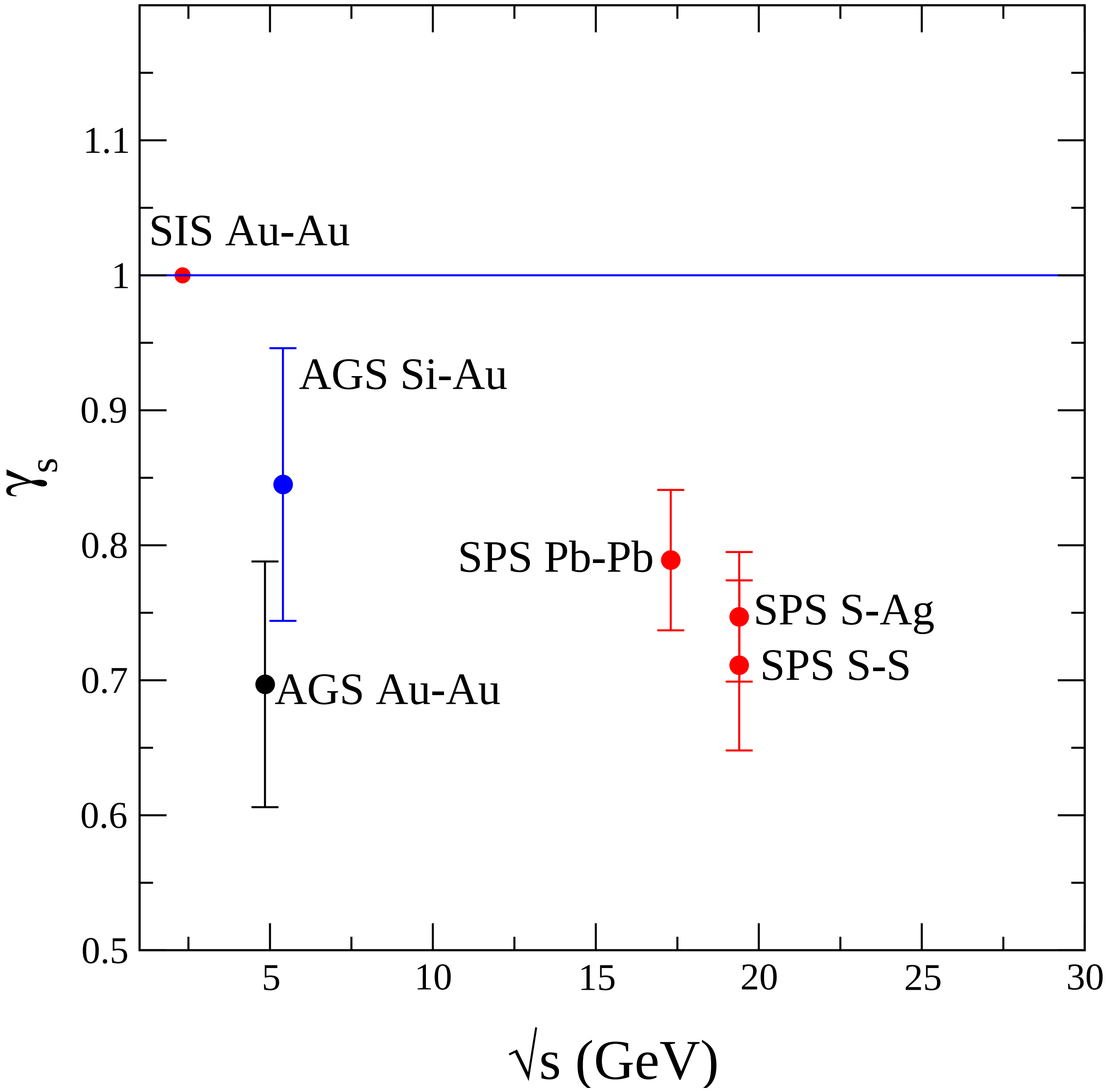


$\bar{\Lambda}/\Lambda$ preliminary new value = 0.10 ± 0.02 (R. Stock, private comm)

χ^2 down to 8.1/6

γ_s seems to be fairly constant but definitely <1

S-S and S-Ag data from: F.B., M. Gazdzicki, J. Sollfrank Eur. Phys. C 5 (1998) 143



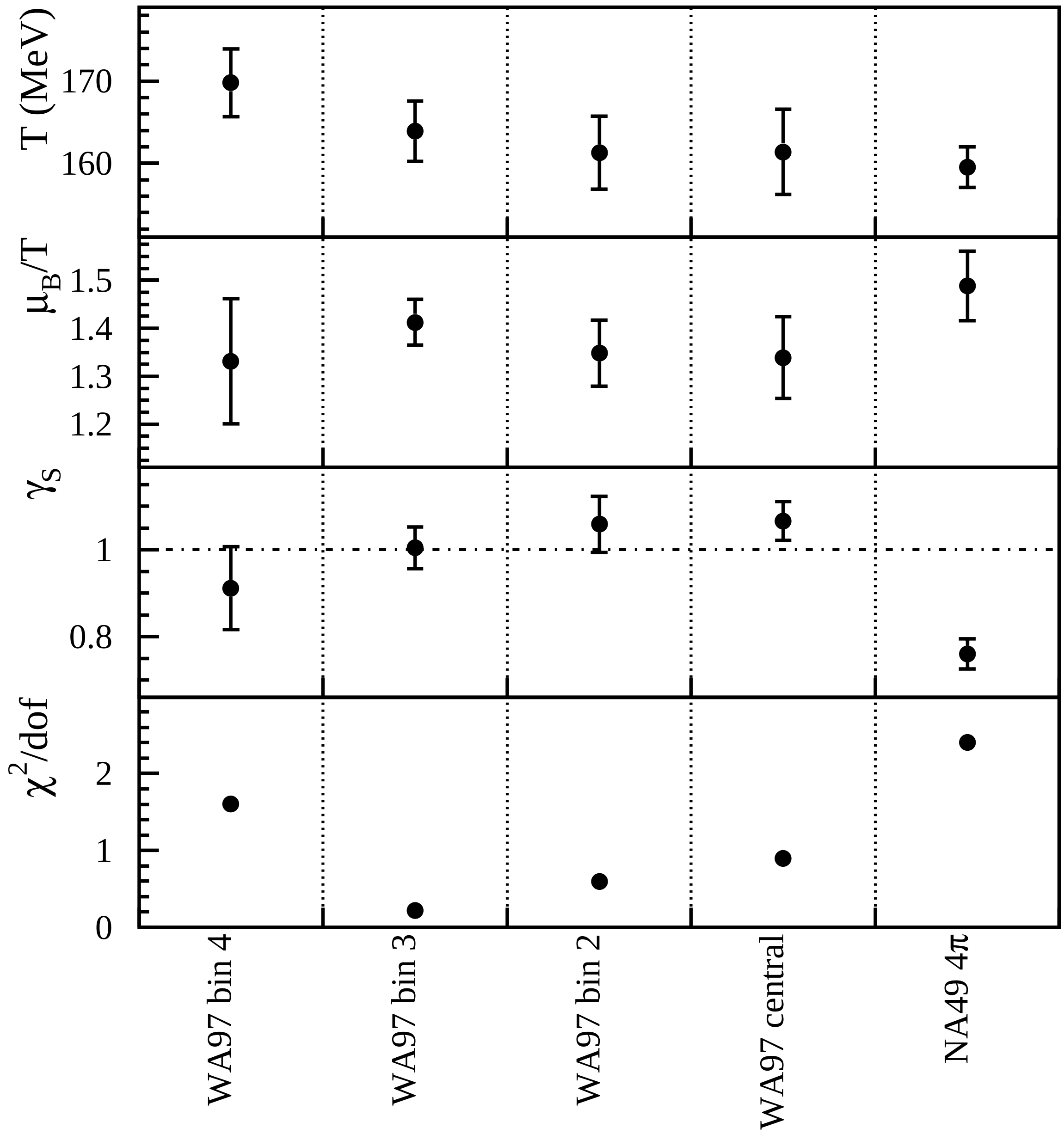
$\gamma_s < 1$ does not depend on ϕ and Ξ

γ_s < 1 does depend on kinematical cuts

4-parameter fit to WA97 7 multiplicities measured in 4 centrality bins (from WA97 web page, data in QM99) constrained with S=0

T ~ 160 MeV

F.B. Trento workshop 2001



FIT TO RHIC RATIOS (preliminary)

F.B. For this workshop

Same data set as in:

P. Braun Munzinger et al., hep-ph 0105229 (9 ratios)

T (MeV)	167.6 ± 7.6	205.4 ± 18.0
μ_B/T	0.270 ± 0.030	0.264 ± 0.028
γ_S	0.962 ± 0.135	0.970 ± 0.138
χ^2/dof	9.8/6	4.2/6
	Weak decays	No weak decays

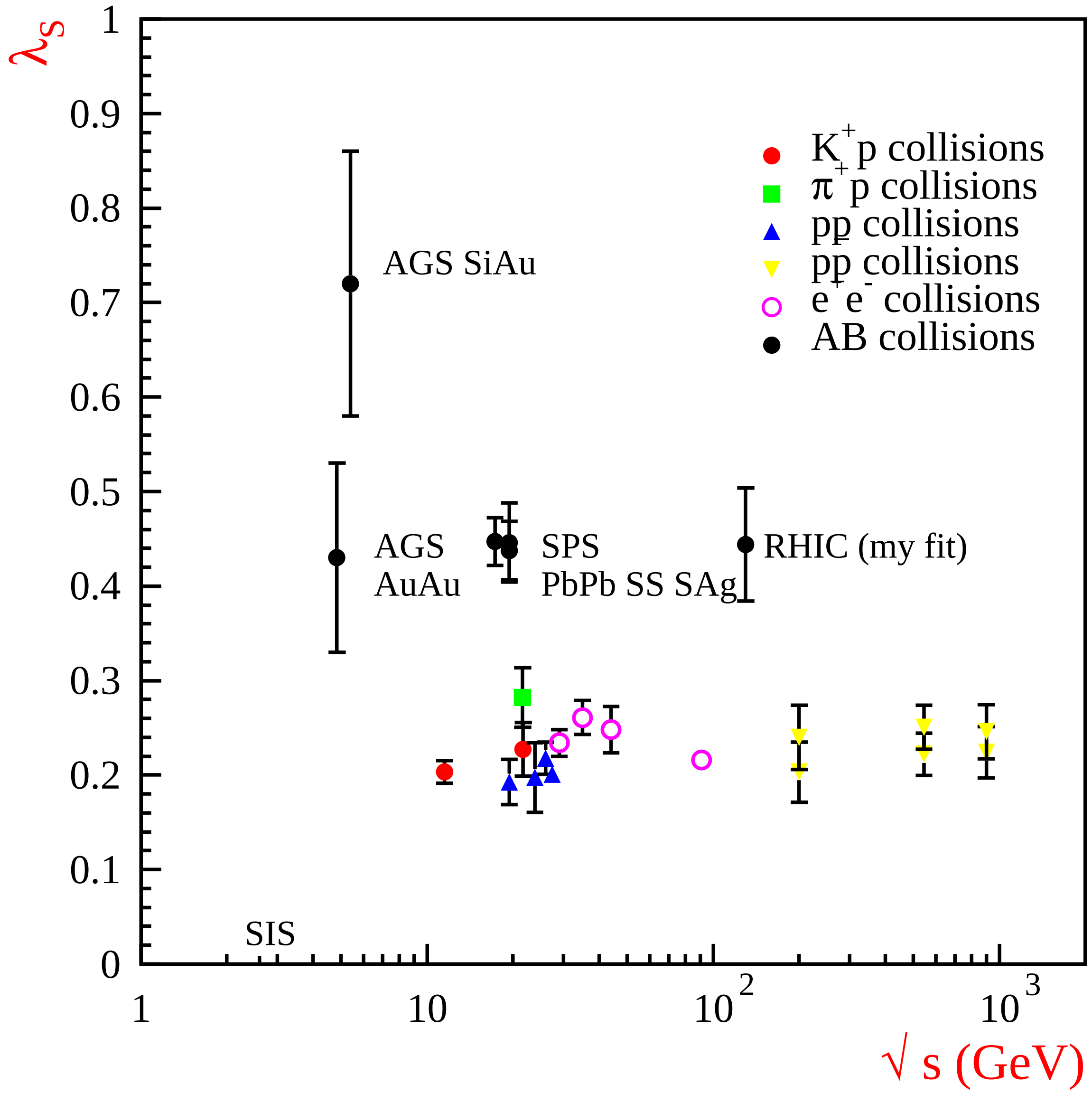
Only the fit with weak decays switched on makes sense.

Results in close agreement with W. Florkowski et al., nucl-th 0106009 and compatible (50% weak decays) with P. Braun Munzinger et al., hep-ph 0105229

The constraint S=0 has been used

λ_s shows a non-trivial behaviour

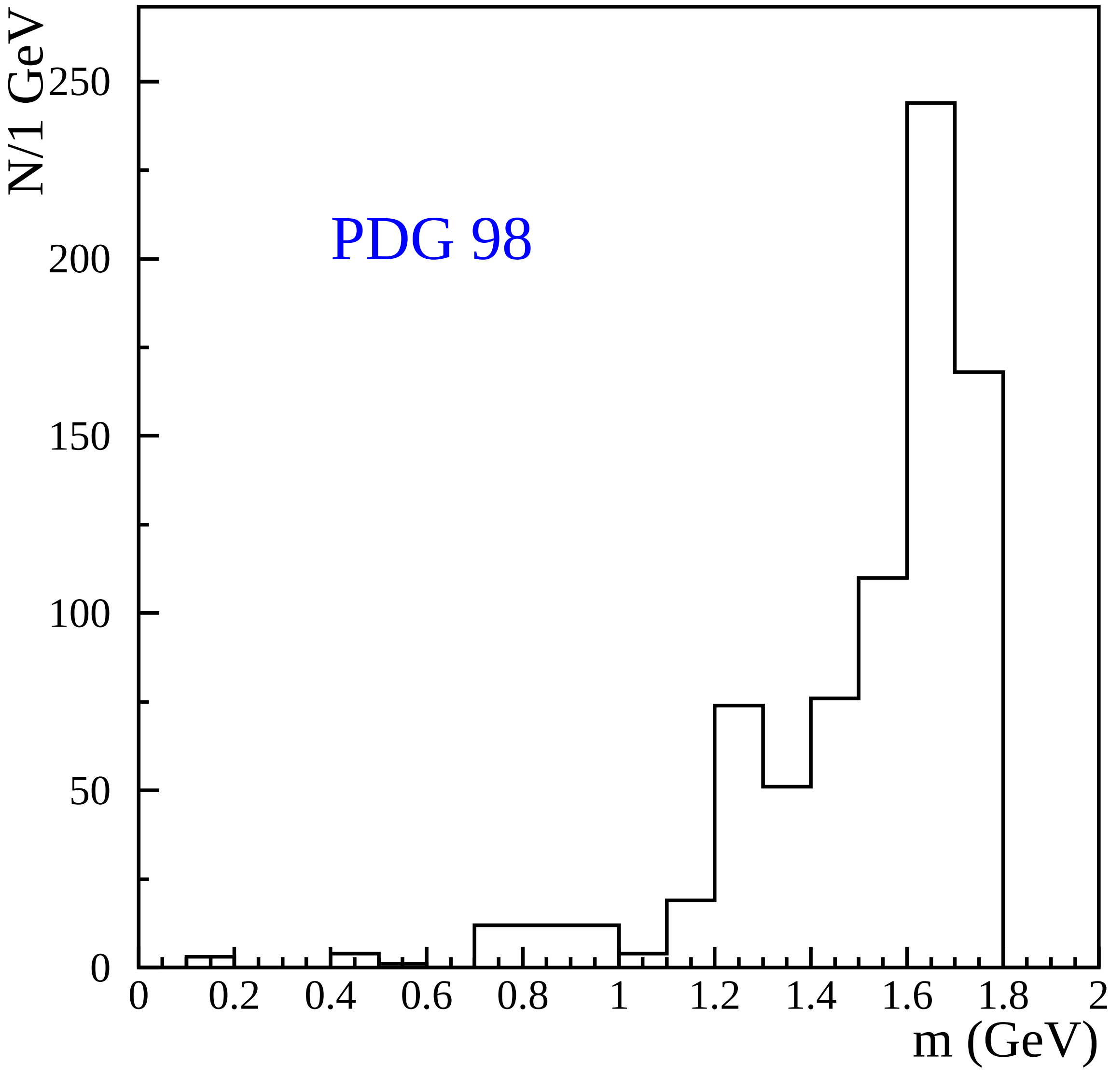
S-S and S-Ag data from: F.B., M. Gazdzicki, J. Sollfrank Eur. Phys. C 5 (1998) 143



TECHNICAL ISSUES

Stability of fit results

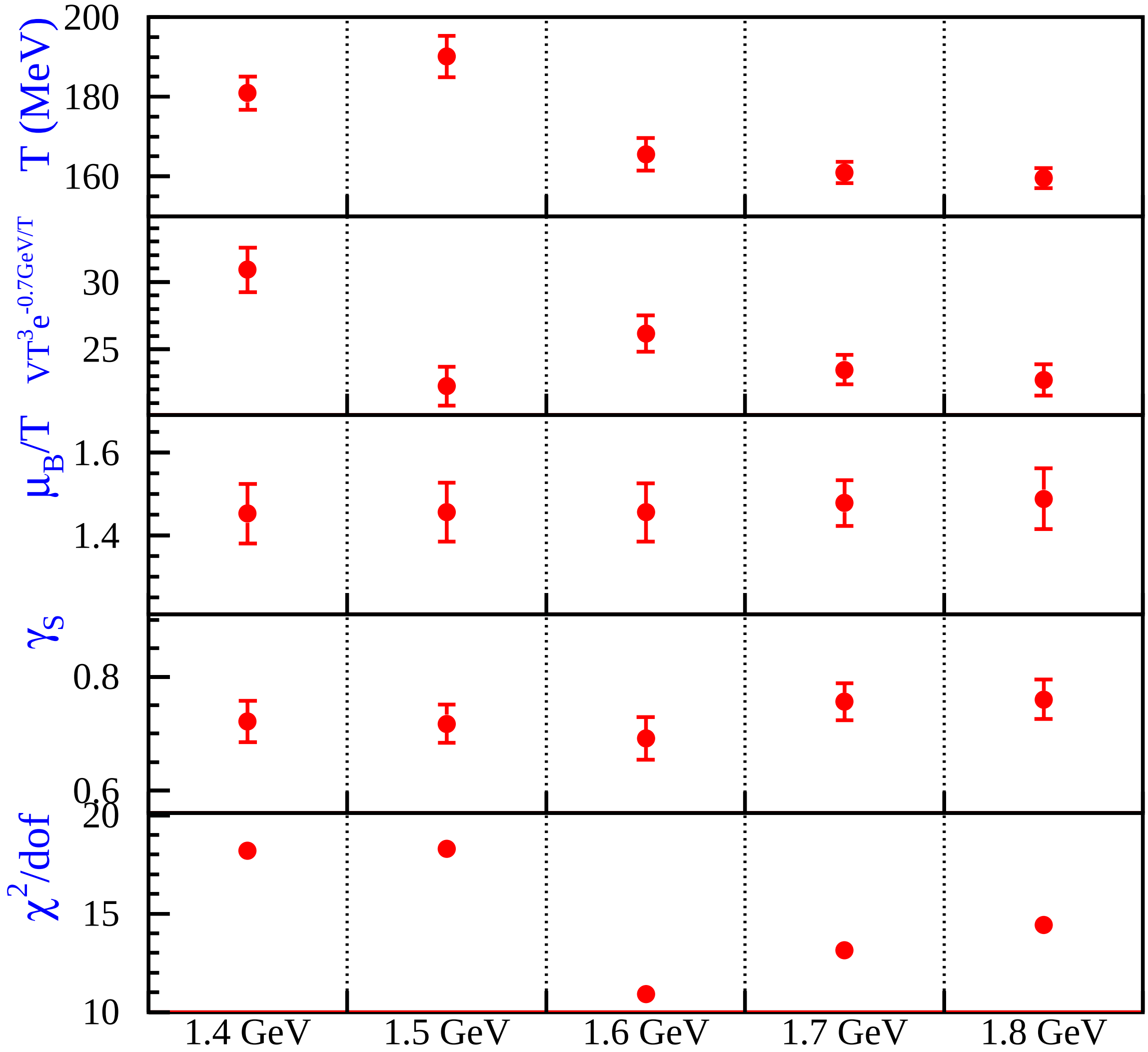
Results of multiplicity fits should be independent of the cut-off on hadron mass spectrum



Stability of fit results II

TEST: move down the cutoff from maximal (e.g. 1.8 GeV) and study the variations of best-fit parameters *and of primary particle multiplicities*

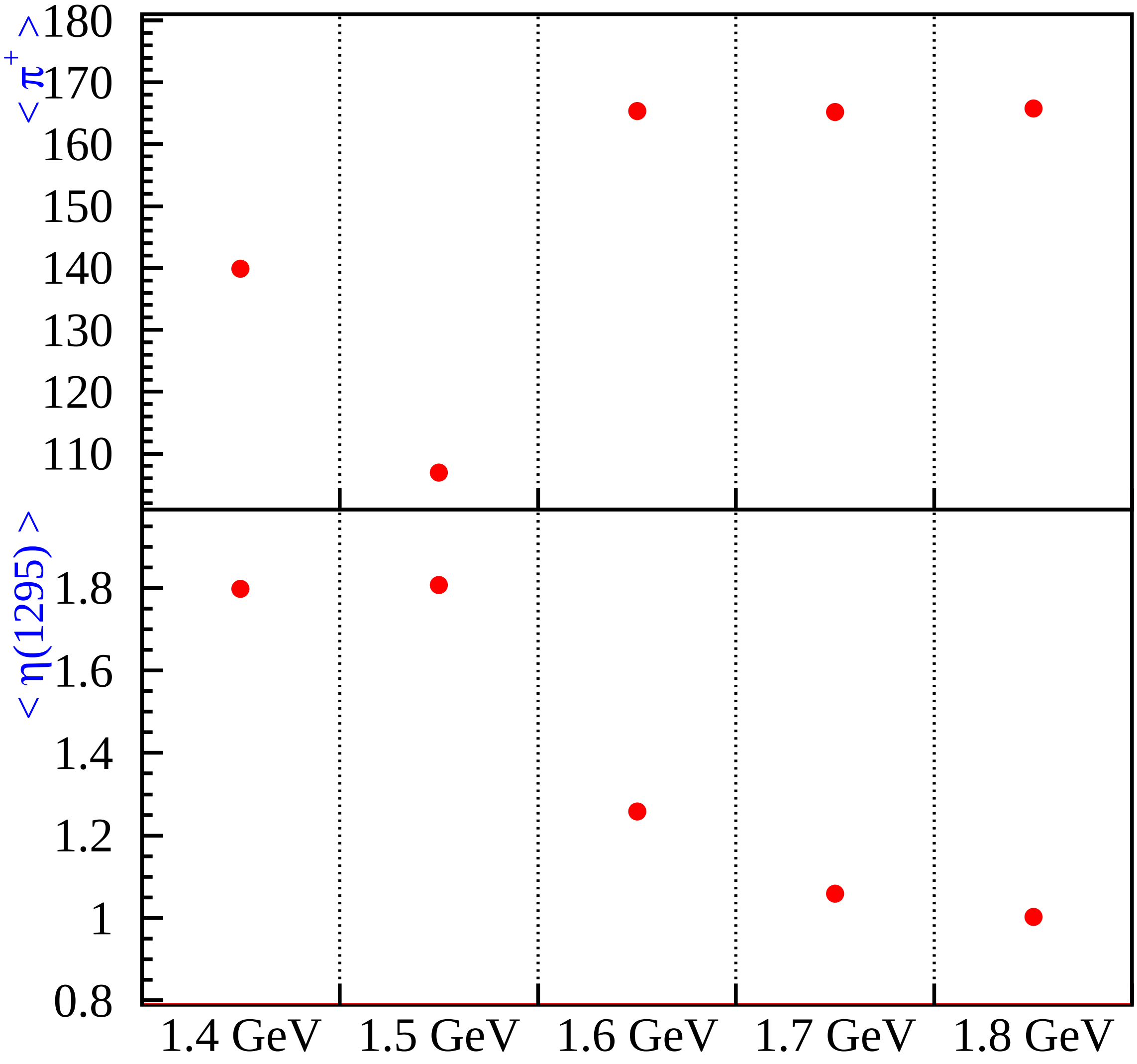
NA49 PbPb data set (4π multiplicities) used in: F. B., J.Cleymans, A. Keranen, E. Suhonen, K. Redlich, Phys. Rev. C 64, 024901, 2001



Stability of fit results III

Study the variation of primary multiplicity of both a low-mass and a high-mass particle in order to assess the stabilisation of the fit

NA49 PbPb data set (4π multiplicities) used in: F. B., J.Cleymans, A. Keranen, E. Suhonen, K. Redlich, Phys. Rev. C 64, 024901, 2001



SOME VARIATIONS

NA49 PbPb data set (4π multiplicities) used in:

F. B., J.Cleymans, A. Keranen, E. Suhonen, K. Redlich, Phys. Rev. C 64, 024901, 2001

T (MeV)	159.5 ± 2.5	159.3 ± 2.4	159	154.0 ± 2.5
μ_B/T	1.489 ± 0.073	1.490 ± 0.073	1.478	1.459 ± 0.066
γ_S	0.760 ± 0.035	0.706 ± 0.034	0.732	0.866 ± 0.043
χ^2/dof	14.4/6	14.3/6	15.7/6	15.1/6
	Main fit	No QS	No BW	Weak decays

SYSTEMATIC ERRORS

Hadron masses, resonance widhts and branching ratios used in the fit are affected by an experimental error

an additional systematic error is involved

EFFECTIVE VARIANCE METHOD:

1) Fit by using only experimental errors on multiplicities

2) With the preliminary fit values, vary the l^{th} hadronic parameter affected by a significant uncertainty and calculate all Δn_i's (i=1,...,number of data points)

3) Calculate the systematic covariance matrix

$$C_{ij}^{sys} = \sum_l \Delta n_i \Delta n_j$$

4) Add systematic to experimental covariance matrix and redo the fit

All m's and Γ's with error > 2‰ + 130 BR's varied !

The inclusion of systematic errors has an impact on fitted parameters only if there are many accurately measured multiplicities (e.g. In e^+e^- or pp) but can be neglected for heavy ion collisions at least up to SPS

CANONICAL VS GRAND-CANONICAL

A. Keranen, Trento workshop June 2001

F. B. and A. Keranen, in preparation:

Numerical integration is feasible (with some tricks) even for B ~ 400

Study for a situation ~ SS collisions @ SPS:

T=180 MeV, γ_S = 0.7, V=3 10^4 GeV^{-4} = 230.5 fm^3

and B=54

π^+	85.87500	86.23700	0.42%
Ξ^-	0.54770	0.56420	2.92%
Ω	0.05334	0.05736	7.01%
	Canonical	G-Canonical	Δ %

Canonical corrections are negligible for PbPb and AuAu systems from AGS onwards.

⚠

The exact strangeness conservation involves suppressions with respect to GC ensemble for Si-Au @ AGS and AuAu @SIS (strangeness canonical ensemble J. Cleymans et al. Phys. Rev. C 57 3317, 1998)

Recommendations and desiderata about papers on particle yields and ratios

- Numbers and tables, not only plots

- Quote errors separately (statistical, systematic and extrapolation)

- Extrapolations to full phase space, if possible, with relevant systematic error (estimated, for instance, by means of different formulae or models)

- Clear and unambiguous statements about weak decay products (e.g. in table captions): what is included, what is not

- Strong preference for weak decay products included either 0% or 100%, no intermediate number

CONCLUSIONS

- In PbPb the fitted temperature with 4π NA49 data is now very close to 160 MeV (our best fit 158.1$\pm$3.2 MeV); this is confirmed by a fit to WA97 data in a limited phase space region in several centrality bins. This value is amazingly close to that obtained in the most accurate fits to e^+e^- and pp collisions (*can this be accidental ?*)

- No complete strangeness equilibrium either in elementary (s/u ~ constant) or HI collisions in full phase space up to SPS: $\gamma_S \sim 0.7$-0.8 <u>independently of the inclusion of doubly strange particles</u>

- Complete strangeness equilibrium: a local property at midrapidity or an artefact of kinematical cuts, as shown with stand-alone WA97 fits, and use of inhomogeneous data? Very important question for RHIC data.

- Crucial issue for the extraction of parameters is how to deal with weak decay products